1900MNRAS..60..485P

The augmentation of $S-s$ for Altitude $41^\circ\ 49'{\cdot}5 = 10''{\cdot}80$

$\therefore$ The augmented value of $S-s = 19''{\cdot}51$

The Parallax in R.A. $= (P-p)\,\rho \cos l \sin t$

$= 35'\ 4''{\cdot}75$

The Parallax in Dec. $= (P-p)\,(\rho \sin l \cos \delta - \rho \cos l \sin \delta \cos t)$

$= 25'\ 28''{\cdot}69$ (above the centre)

The Soli-lunar position in R.A. $= (A-a) \cos D$

$= 34'\ 53''{\cdot}85$ (E. of Minor Axis of parallactic ellipse)

" " " in Dec. $= D-\delta$

$= 25'\ 24''{\cdot}46.$

To this $4''{\cdot}20$ must be added, on account of the lunar orbit, making the position $25'\ 28''{\cdot}66$ above the centre.

The extent of motion of parallax in 1^m in R.A.

$$= (P-p)\,\rho \cos l \cos t\ \Delta t = 7''{\cdot}02.$$

The extent in Dec. $= (P-p)\,\rho \cos l \sin \delta \sin t\ \Delta t.$

Therefore the tangent of the angle at the parallax point

$$= \sin \delta \tan t = 25^\circ\ 35'.$$

The Soli-lunar motion in R.A. in $1^m = (2^s{\cdot}484 - 0^s{\cdot}178) \cos \delta$

$= 32''{\cdot}24$

" " " in Dec. in $1^m = 2''{\cdot}529 - 0''{\cdot}405$

$= 2''{\cdot}124.$

The diagram (Plate 14) is constructed from these elements: P is the parallax at $4^h\ 2^m$ and its line of motion is drawn. M is the Soli-lunar centre and its line of motion is drawn. The diagram shows that the second contact occurs $20^s\frac{1}{4}$ before $4^h\ 2^m$, and the third contact $1^m\ 12^s\frac{1}{2}$ after it.

An Application of Projective Geometry to Binary Star Orbits.

By H. C. Plummer.

1. In the numerous methods which have been employed in the determination of the orbits of double stars, it is natural, on account of the comparatively coarse nature of the observations, that much use has been made of graphical devices. These for the most part are of the "trial and error" kind. T. N. Thiele* and J. M. Wilson,† however, have given direct geometrical constructions by which the relations between the apparent and the

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* *A.N.* 1227.

† *M.N.* xxxiii. p. 375.

1900MNRAS..60..486P

real orbit are exhibited, and the former has made practical use of his method. It is my object here to deduce these relations from a consideration of the projective properties of the ellipse, and to show how the geometrical elements of the orbit, viz. i, Ω, λ, e, a, may be determined by a tolerably simple construction. It is, of course, impossible to use direct time measures in a method which is essentially geometrical. As in Savary's * first method, the necessary data are taken in the shape of five positions in the apparent orbit. Then the problem to be solved may be stated thus: Given five points and a certain fixed point, to find the orthogonal projection such that the corresponding five points will lie on an ellipse the focus of which projects into the fixed point. The solution takes the form of constructing the line of nodes, finding the inclination of the orbit, and laying down the five points corresponding to the apparent positions. The ruler and compasses alone are used in the process, and it is quite unnecessary to draw an ellipse.

2. It is convenient to consider in the first place a fundamental problem the solution of which will be required. This is to find the orthogonal projection by which two right angles project into two given overlapping angles contained by four lines which meet in a point. Let S be the point (Plate 15, fig. 1), and suppose that the lines intersect a transversal in AA′, BB′. On AA′, BB′ as diameters describe circles meeting in a point F. Then if F be rotated about AA′ out of the plane of the paper, the given figure may be considered as a projection on to the plane SAA′ from the point at infinity on the line SF as vertex. The projection is a parallel one, and is in addition orthogonal if SF be perpendicular to the plane SAA′. Hence SF must be perpendicular to AA′. Now if N is the foot of the perpendicular from F on AA′, $AN.NA'=NF^2=BN.NB'$, or N is the centre of the involution range determined by AA′, BB′. Hence SN and the line through S parallel to AA′ are a corresponding pair of the involution pencil S (AA′, BB′,...), since N being the centre of the range is the point corresponding to the point at infinity on AA′. Conversely if the transversal AA′ be drawn parallel to one line of the orthogonal pair in the involution pencil S (AA′, BB′...), the point F constructed as above will lie on the perpendicular from S to the transversal. Then when F is rotated about AA′ till FS is perpendicular to the plane SAA′, the projection is orthogonal, and we have the projection required, for the angles ASA′, BSB′ are the projections respectively of AFA′, BFB′, which being angles in semicircles are right angles. We thus have a simple method suggested for constructing the transversal or turning line AA′ through any fixed point C, which is as follows. Describe a circle on SC as diameter and let it cut the rays of the pencil S (AA′, BB′) in A_1, A_1', B_1, B_1'. Join OK, where K is the centre of the circle and O is the intersection of

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* *Conn. des Temps,* 1830.

1900MNRAS..60..486P

Fig. 1

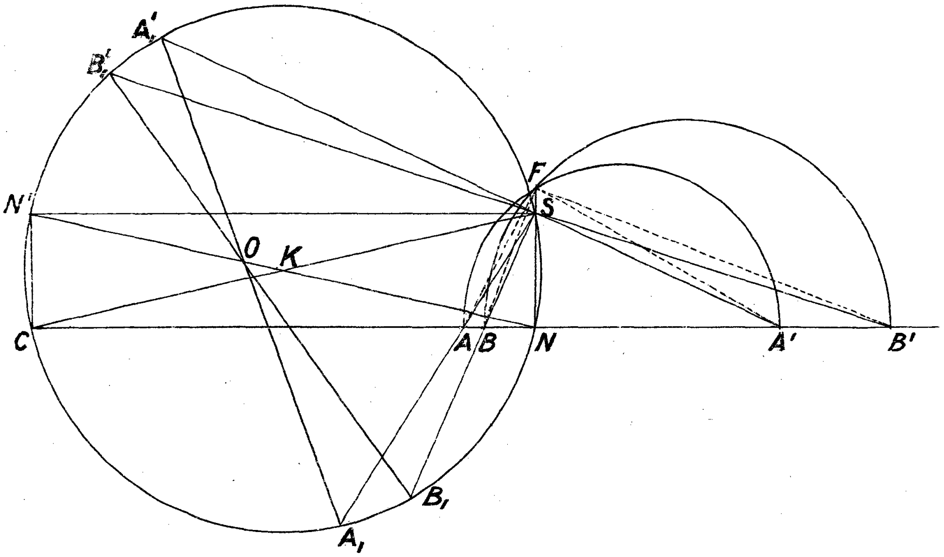
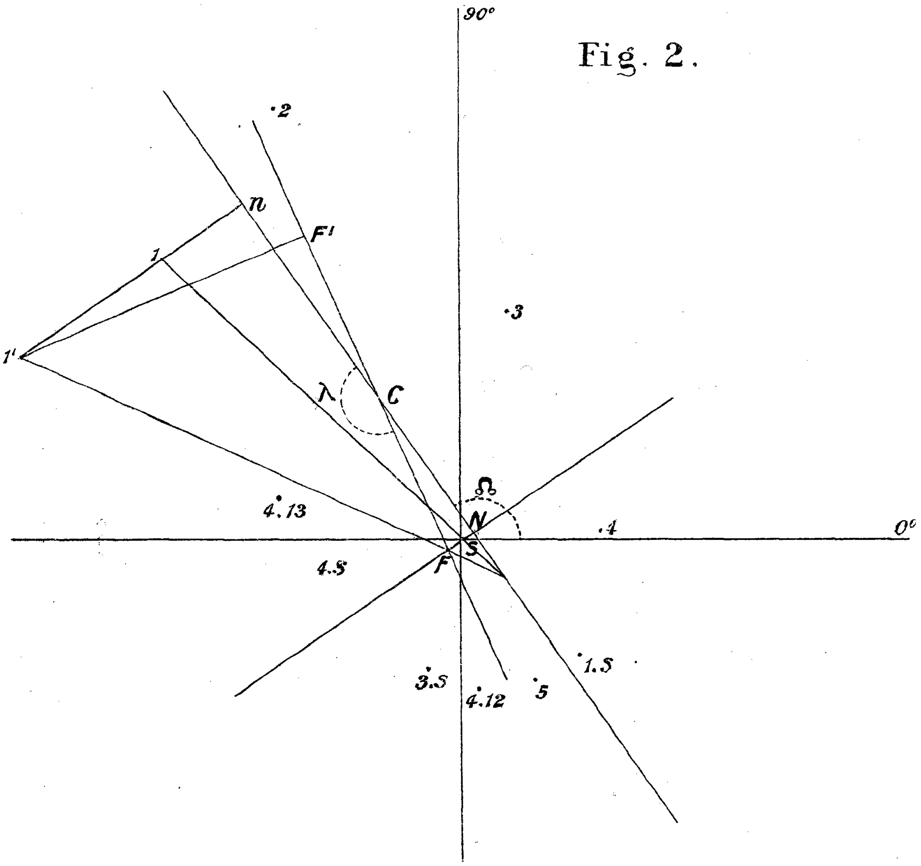


Fig. 2.



1900MNRAS..60..486P

A_1A_1' and B_1B_1', and let OK meet the circle in NN′. Then SN, SN′ are orthogonal rays of the involution, since NN′ is a diameter and NN′, A_1A_1', B_1B_1' are concurrent chords. Also clearly CN, CN′ are parallel to SN′, SN respectively. Finally the choice between CN and CN′ as a transversal must be decided by the consideration that it must fall across the obtuse angle between SA and SA′, for otherwise the point F would lie nearer the turning line than S, and the orthogonal projection would become impossible.

3. The general method of procedure is now obvious. Through S, the position of the principal star, two pairs of conjugate lines must be drawn. If then through C, the centre of the apparent ellipse, a line be drawn by the construction given at the end of § 2 parallel to one of the orthogonal pair of the involution of conjugate lines through S, this line is the turning line required, or the line of nodes. Then the point F is found as in § 2, and this when the plane FAA′ is rotated so that FS is perpendicular to the plane SAA′ projects orthogonally into the point S. Moreover, since the conjugate pairs of lines through S are the projections of orthogonal pairs through F, F is the focus of the ellipse which projects into the apparent orbit. As the line of nodes and the focus of the true orbit are now known, the problem in view has been virtually solved. If the method of Sir John Herschel has been followed as far as the drawing of the apparent ellipse, the centre C is known and pairs of conjugate lines may be constructed by drawing chords through S, constructing tangents to the ellipse at their extremities and joining S to the poles of the chords. The completion of the solution is then simple and rapid.

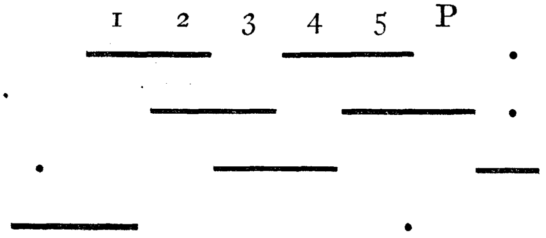
4. But the actual drawing of the apparent orbit is unnecessary and even objectionable. For it must be a more or less troublesome and unsatisfactory process, and besides the choice of five characteristic points should be not merely easier but also capable of at least equivalent accuracy. Hitherto observed distances have been utilised only in a minor degree in the calculation of orbits, having been long under the ban of Sir John Herschel's authority. Even at the time when this neglect may have been justified by the inferiority in the measures of this coordinate, Encke held other views as to their value. But now that greatly increased accuracy is attainable in this part of the observations, it seems permissible to place reliance on this material, and perfectly feasible to employ it for the purpose we have in view.

The five points will be denoted by the numbers 1, 2, 3, 4, 5 (Plate 15, fig. 2). Also the other extremity of the chord through 1 parallel to the line joining 2 and 3, for instance, will be denoted by the notation 1.23, while the other extremity of the chord through 1 and S will be denoted by 1.S. Now points such as 1.23 and 1.S can be easily found by the well known construction with the ruler only. Five of these points are required, two of the former type and three of the latter. The two will determine two pairs

1900MNRAS..60..486P

of parallel chords, and will therefore give the centre of the ellipse as the intersection of the two diameters which bisect the parallel chords. The other three points give three chords which meet in S, and by taking these two at a time we obtain three quadrangles inscribed in the conic, for each of which S is a harmonic point. Thus, for instance, if the quadrangle 1, 2, 1.S, 2.S, be taken, and the lines joining 1, 2 and 1.S, 2.S, meet in Z, while the lines joining 1, 2.S and 2, 1.S meet in Z′, the pair of lines SZ, SZ′ are conjugate. Three pairs of conjugate lines can therefore be drawn, and as two only are required those may be chosen which can be constructed most easily.

5. The construction of the five additional points is an important part of the method, and as it is rather complicated I give the following plan which I have found a convenient aid in order to avoid confusion. Suppose that the point P is required in which a fixed line through 5 cuts the ellipse again. Then the following scheme may be useful :—



in which each line indicates that the points to be found vertically above it are to be joined, and which is to be taken line by line, and read thus : (1) join 1, 2 and 4, 5 ; these meet in a point (placed on the right) ; (2) join 2, 3 which meets the line 5P in a point (also placed on the right) ; (3) join 3, 4 and the two points of intersection ; these lines meet in a point (placed on the left) ; (4) join the latter point to 1, and this line intersects 5P in the point required. The scheme can be followed mentally on the figure before the construction is made. In this way it can be decided first whether the intersections will fall conveniently on the paper. If, as will often happen, they fall outside it, it will be better to alter the order of the first four points in the scheme. There will then be a new hexagon, and the Pascal line may be constructed more easily. For instance, if the intersection of 12 with 45 is near the centre of the paper, and the intersection of 23 with 5P lies beyond the edge, it is natural to start with the hexagon 2 1 3 4 5 P instead of 1 2 3 4 5 P. Such details will naturally occur, and are only mentioned here because they have considerable importance in practice.

6. For the sake of clearness it seems well to summarise the actual steps which have to be made, referring for greater definiteness to an actual case. The natural order will be :

(*a*) To select the characteristic positions and to plot the five places to a suitable scale.

1900MNRAS..60..486P

(*b*) To construct the points 4.12 and 4.13, to draw the diameters bisecting the parallel chords, and so to find the centre C.

(*c*) To construct the points 1.S, 3.S, and 4.S, to complete the quadrangles 1, 4, 1.S, 4.S, and 3, 4, 3.S, 4.S, and by means of them to draw two pairs of conjugate lines through S.

(*d*) To construct, as explained in § 2, the orthogonal pair of the conjugate involution, and to draw through the centre the line of nodes.

(*e*) To find the point F as in § 2; this is the focus of the real orbit. Also it is evident that the inclination is given by

$$\cos i = \mathrm{SN}/\mathrm{FN}.$$

(*f*) To lay down the five positions in the real orbit, and to deduce the values of the elements a, e and λ. This will be next explained.

7. In the first place CF is the line of apsides, and CN is the line of nodes; it follows that $\pi-\lambda$ is the angle FCN. As the relations between the planes of the apparent and real orbits have been already found, it is unnecessary to retain the distinction. In fact it is simpler to lay down the five real positions in the plane of the paper by considering them as in parallel homology with the apparent figure. The line of nodes CN is the axis of homology, the centre of homology is at infinity on a line at right angles to CN, and F and S are a pair of corresponding points. The point 1′ corresponding to 1 is then easily found, since 11′ is perpendicular to CN, and 1′F and 1S meet on CN. Similarly the homologues of the points 2, 3, 4, 5 can be laid down. Then if F′ be the second focus, five measures of a are available, viz. $\frac{1}{2}$ (1′F + 1′F′), &c. But CF $= ae$, and so the eccentricity is found by dividing CF by the mean value of a. Thus all the geometrical elements of the orbit have been found, and finally the two remaining elements, the mean motion and the epoch, can be calculated at once from the dates of two observations, though of course better from more extensive data. It is to be remarked that the agreement in the values of a, found as above from the five points, is a measure of the correctness of the drawing, on which it forms a useful check. On the other hand, additional normal places may be laid down, and their homologues may be constructed. From these an additional set of values of a can be obtained. The agreement of these values among themselves, and with the former set, furnishes a criterion of the concordance of the observations involved, and of the accuracy of the assumption that the five adopted positions characterise the orbit.

8. Having now sketched a method of dealing with binary star orbits, it is interesting to consider the result of applying it to a particular system. In order best to test the practicability of the method and to judge the order of accuracy which it is capable of giving, I chose the star 70 *Ophiuchi*, because, apart from the

1900MNRAS..60..486P

historic interest which the system possesses, and the fact that sufficient observations existed to make a fair trial possible, it has been made the subject of an exhaustive research by Professor Schur, and therefore its orbit is to be considered so far definitive as the refinements of rigorous calculation could render it. The five characteristic places were chosen at random, but so as to cover as large an arc as possible, though it was judged advisable to leave on one side the earlier observations of Sir W. Herschel and others. The following are taken from a list of normal places given by Dr. See * :—

No.	Date.	Pos. Ang.	Dist.
1	1830·57	136°·8	5″·58
2	1854·48	113·2	6·37
3	1877·60	77·4	3·23
4	1887·68	3·8	1·91
5	1895·64	296·1	2·14

These were plotted to the scale 1″=22 mm., and their relative positions are quite roughly indicated in fig. 2. The result of measuring the drawing was to give the following elements, against which I place Professor Schur's † elements for comparison :—

		Schur.
i	57° 40′	60°·08
☊	121 49	121·31
λ	167 2	168·3
a	4″·596	4″·60
e	·515	·4751

The value of *a* given above rests on a single measure of $F1'+F'1'$. This in itself is an indication that the above elements are really what might not unfairly be called a rough preliminary measurement, the apparent accuracy being altogether unexpected and surprising. They represent such results as may reasonably be attained in an hour's work after the five points have been selected and laid down.

9. It then became a matter of interest to calculate the epoch and the mean motion. Using the extreme places 1 and 5, we get these values :—

		Schur.
T	1895·445	1896·4661
P	86·8	88·3954

Instead of proceeding with these, which evidently rest on an insufficient basis, it seemed better to use additional places, and the following elements were adopted, though it must be under-

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* *Ast. Jour.* 363.

† *A.N.* 3220–1.

1900MNRAS..60..486P

stood that they were not obtained, as they might have been, from a least squares solution :—

$$T = 1895{\cdot}70,\quad P = 88{\cdot}32,\quad a = 4''{\cdot}495.$$

The following table exhibits the comparison between the places computed from these elements and the observed coordinates, which have been taken from Dr. See's paper already quoted, except the last three places depending solely on Mr. Maw's measures.*

Date.	Position Angle.	Dis- tance.	C—O Angle.	C—O Distance.
	°	″	°	″
1830·57	136·8	5·58	−1·3	−0·27
1831·58	135·1	5·68	−0·9	−0·18
1832·62	133·4	5·75	−0·5	−0·09
1833·42	132·8	6·14	−0·9	−0·37
1834·55	130·8	6·04	−0·1	−0·12
1835·60	130·7	6·11	−1·1	−0·06
1836·52	128·9	6·34	−0·2	−0·20
1837·64	127·9	6·45	−0·3	−0·19
1838·59	126·6	6·64	0·0	−0·29
1839·58	125·5	6·66	+0·2	−0·24
1840·47	126·5	6·38	−1·7	+0·11
1841·64	124·0	6·60	−0·2	−0·04
1842·57	123·8	6·57	−0·9	+0·06
1842·60	123·5	6·79	−0·6	−0·18
1843·55	122·1	6·57	+0·1	+0·10
1844·44	121·3	6·66	+0·1	+0·02
1845·48	120·9	6·64	−0·3	+0·07
1846·46	119·3	6·76	+0·4	−0·01
1847·47	119·1	6·85	−0·3	−0·11
1848·38	118·4	6·81	−0·3	−0·06
1849·39	118·1	6·64	−0·8	+0·10
1850·55	116·4	6·78	−0·1	−0·06
1851·54	115·5	6·57	0·0	+0·11
1852·69	114·9	6·56	−0·4	+0·09
1853·57	114·9	6·42	−1·1	+0·19
1854·48	113·2	6·37	−0·3	+0·20
1855·52	113·3	6·45	−1·3	+0·06
1856·43	111·7	6·34	−0·5	+0·10
1857·52	111·0	6·31	−0·8	+0·07
1858·39	109·1	6·01	+0·3	+0·29
1859·66	108·4	6·24	−0·3	−0·05
1860·70	107·3	6·33	−0·2	−0·24
1861·67	106·2	5·70	−0·1	+0·20
1862·59	105·6	5·83	−0·4	+0·05
1863·54	104·8	5·62	−0·6	+0·14
1864·54	104·1	5·43	−1·0	+0·20
1865·50	102·4	5·32	−0·5	+0·18
1866·43	101·2	5·31	−0·6	+0·06
1867·50	99·6	5·18	−0·5	+0·03
1868·65	98·6	4·90	−1·1	+0·14
1869·80	96·7	4·64	−1·0	+0·21
1870·51	94·2	4·52	+0·3	+0·20
1871·56	93·4	4·34	−0·6	+0·19
1872·51	91·6	4·20	−0·7	+0·16
1873·56	88·1	4·01	+0·7	+0·15
1874·61	87·7	3·81	−1·4	+0·14
1875·61	84·2	3·59	−0·5	+0·15
1876·57	80·7	3·48	+0·3	+0·06
1877·60	77·4	3·23	+0·4	+0·08
1878·58	74·3	3·05	0·0	+0·05
1879·57	69·5	2·95	+0·8	−0·05
1880·59	64·0	2·64	+1·4	+0·04
1881·56	60·3	2·55	−0·3	−0·05
1882·60	52·5	2·48	+0·9	−0·17
1883·62	44·0	2·31	+2·0	−0·16
1884·56	36·0	2·17	+2·3	−0·13
1885·61	25·9	2·06	+2·6	−0·12
1886·61	14·3	1·93	+4·1	−0·01

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* *Mem. R.A.S.* liii.

1900MNRAS..60..486P

Date.	Position Angle.	Distance.	C—O Angle.	C—O Distance.
	°	″	°	″
1887·68	3·8	1·91	+2·8	−0·01
1888·62	353·9	2·11	+2·5	−0·21
1889·53	345·9	2·08	+1·1	−0·10
1890·57	336·7	2·21	+0·2	−0·17
1891·59	327·4	2·23	+0·7	−0·11
1892·52	320·8	2·26	−0·2	−0·09
1893·62	312·9	2·25	−1·2	−0·05
1894·69	304·2	2·30	−0·7	−0·09
1895·32	298·6	2·22	−0·1	−0·05
1895·64	296·1	2·14	−0·1	+0·02
1896·61	289·0	2·31	−1·3	−0·26
1897·55	279·7	1·80	−1·0	+0·10
1898·55	270·7	1·91	−2·4	−0·07

The fact must be insisted on that no endeavour has been made to give a definitive orbit to 70 *Ophiuchi*, and nothing more has been attempted than to give a practical illustration of the working of the general method. Herein lies the justification for the merely rough adjustment of the elements T, P, and a. The other elements used are those given above, and as regards these an improved value might probably be assigned to the inclination. The value given rests on the measurement $SN = 6·15$ mm., $FN = 11·5$ mm. Greater relative accuracy can be attained in the measurement of longer lines, and it is therefore better to rely on the equivalent ratio $1n/1'n$ (fig. 2), for which, on reference to the original drawing, I find $1n = 31·3$ mm., $1'n = 60·0$ mm. These measurements give

$$i = 58^\circ 33'.$$

1900 *April* 10.

On the Orbit of β 883. By T. Lewis.

The uncertainty in the period of this double star arises in the first instance from the difficulty in fixing the proper quadrant owing to the components being of nearly equal magnitude; and secondly, by reason of their closeness, the distance between the two stars never exceeding 0″·3.

Dr. See has assigned a period of 5½ years, which makes β 883 the most rapid binary known. On the other hand, Professor Glasenapp's period of 16·8 years would merely rank it amongst the most rapid pairs.

Since 1896 this star has been under almost continuous observation, and hence the area swept out during these four years may be taken as a standard area affected with but small error. Working backwards with this argument, it became at once evident that Burnham's measure in 1891·97 really belonged to the third quadrant. Admitting this, we must place the observations of 1894 and 1895 in the opposite quadrants set down by the observer. These are all the changes requisite, and are shown in the following means by brackets :—